COEFFICIENT ESTIMATES CERTAIN SUBCLASSES OF ANALYTIC AND BI-UNIVALENT FUNCTIONS INVOLVING FOX WRIGHT AND KUROKI-OWA

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Abstract

Let $K_{\Omega,\Sigma}^{\theta}\left(\lambda;\alpha;\beta\right)$ *a new subclass bi-univalent involving Fox-Wright functions such that* f *and it inverse* f^{-1} *both belong to the class* $K_{\Omega,\Sigma}^{\theta}\left(\lambda;\alpha;\beta\right)$ *defined in the open unit disc* $U=\left\{z\in\mathbb{C}:\left|z\right|<1\right\}$. *In this paper we establish bounds for the coefficients of the functions in the class* $K_{\Omega,\Sigma}^{\theta}\left(\lambda;\alpha;\beta\right)$. *Relevant connections with earlier known results are made.*

Keywords: bi-univalent functions, Fox-Wright functions, analytic functions, Kuroki-Owa functions.

1 INTRODUCTION

Complex analysis is the branch of mathematical that investigates functions of complex numbers. It is useful in many branches of mathematics, such as algebraic geometry, number theory, analytic combinatorics, applied mathematics. Furthermore, in physics and engineering fields, the applications of complex analysis also including the branches of hydrodynamics, thermodynamics, and particularly quantum mechanics, nuclear, aerospace, mechanical and electrical engineering.

In complex analysis, the theory of conformal mappings, has many physical applications and analytic number theory. Complex analysis has become very prominent through complex dynamics and the pictures of fractals produced by iterating holomorphic functions.

Let x and y be real number and let i denote the imaginary unit having the property that $i^2=-1$. A complex number is an expression of the form $z=x+iy$. Write $\mathbb{C}$ to denote the set of complex number. That is
$$\mathbb{C}=\left\{z=x+iy:x,y\in\mathbb{R}\right\}.$$

The theory of analytic functions and more specific univalent functions is one of the most fascinating topics in one complex variable. There are many remarkable theorems dealing with extremal problems for the class S of normalized univalent functions on the unit disk $U=\left\{z\in\mathbb{C}:\left|z\right|<1\right\}$, from the Bieberbach conjecture, which was solved by de Branges in 1985, to others of a purely geometrical in nature. A great variety of methods were developed to study these problems.

The univalent function theory is a fascinating area of research with continued interest in recent times also. This is classified under the broader area of geometric function theory due to the interplay between analysis and geometry.

As a differentiable function of a complex variable is equal to the sum of its Taylor series (that is, it is analytic), complex analysis is particularly concerned with analytic functions of a complex variable (that is, holomorphic functions).

Let A denote the class of the functions of the form:

$$f(z)=z+\sum_{n=2}^{\infty}a_n z^n, \qquad (1)$$

which are analytic in the open unit disk U and normalized by the conditions $f(0)=0$. Let S be the class of functions in A which are univalent in U.

A function $f\in A$ is bi-univalent in U if both f and f^{-1} are univalent in U. Let Σ denote the class of bi-univalent functions defined in the open unit disk U. Recently, the bounds of coefficients of analytic and bi-univalent functions have been studied by many authors.

We refer the reader to Ali et. al. (2012), Caglar et. al. (2013), Obradovic and Ponnusamy (2013), Srivastava et. al. (2013), Ul-Haq et. al. (2014), Pauzi et.

al. (2015), Altinkaya, Ş., & Yalcin, S. (2016) and Bohra, N., & Ravichandran, V. (2017) for recent investigations in this topic.

It is well known that every function $f \in S$ of the form (1) has an inverse f^{-1}, defined by $f^{-1}\left(f(z)\right)=z$ $\left(z \in U\right)$ and $f\left(f^{-1}(w)\right)=w,\ |w|<r;\ r \geq 1/4$, where

$$f^{-1}(w)=w-a_2w^2+\left(2a_2^2-a_3\right)w^3 -\left(5a_2^2-5a_2a_3+a_4\right)w^4+\ldots$$

For two analytic functions f and g in U, we say that f is subordinate to g in U, and write $f \prec g$ if $f(z)=g\left(w(z)\right),\left(z \in U\right)$ for some analytic function $w(z)$ such that $w(0)=0$ and $\left|w(z)\right|<1$ $\left(z \in U\right)$. If g is univalent in U, then the subordination $f \prec g$ is equivalent to $f(0)=g(0)$ and $f(U) \subset g(U)$.

A function $f \in A$ is said to be starlike of order α (0 ≤ α < 1), if it satisfies the condition

$$\Re\left(\frac{zf'(z)}{f(z)}\right)>\alpha,\left(z \in U\right),$$

denoted by $S^*(\alpha)$ is the class of starlike functions of order α. Also, denote $\mathrm{M}(\beta)$ be the subclass of A consisting of functions $f(z)$ which satisfy the inequality

$$\Re\left(\frac{zf'(z)}{f(z)}\right)<\beta,\left(z \in U\right),$$

for some $\beta>1$. Moreover, the subclass $S^*(\alpha,\beta) \subset A$ consists of functions, which satisfy the following inequality

$$\alpha<\Re\left(\frac{zf'(z)}{f(z)}\right)<\beta,\ \left(0 \leq \alpha<1<\beta; z \in U\right)$$

By using hadamard product between the generalized Polylogarithm defined by Al-Saqsi and Darus (2008) and the Fox-Wright generalization defined by Wright (1940). Pauzi et. al (2015) introduce operator involving Fox-Wright function

$$I_{m,k}^{\lambda}(\alpha_1,\beta_1)f(z)=z+\sum_{k=2}^{\infty}\Omega_k^m\Theta_k^m a_k z^k.$$

where

$$\Omega_k^m=\left[\left(\frac{\prod_{j=1}^{q}\Gamma\left(\alpha_j+A_j(k-1)\right)}{\prod_{j=1}^{s}\Gamma\left(\beta_j+B_j(k-1)\right)}\right)\left(\frac{1}{(k-1)!}\right)\right]$$

And

$$\Theta_k^m=\left[\frac{(k-1)^k(k+\mu-2)!}{\mu!(k-2)!}\right],$$

$m \in \mathbb{N}_0=\mathbb{N}\cup\{0\},\ k \geq 2,\ \mu \geq 0.$

The functions classes $M(\beta)$ and $S^*(\alpha,\beta)$ were first investigated by Uralegaddi et al. (1994) and Kuroki and Owa (2011) respectively.

Next we consider the following new subclasses of A.

Definition 1.1: Let λ,α and β be real numbers such that $\lambda \geq 0$ and $0 \leq \alpha<1<\beta$. A function $f \in A$ belongs to the class $K_{\Omega}^{\theta}\left(\lambda;\alpha;\beta\right)$ if f satisfies the inequality:

$$\alpha<\Re\left\{\frac{z\left[I_{mk}^{\mu}(\alpha,\beta)f(z)\right]'}{I_{mk}^{\mu}(\alpha,\beta)f(z)}+\lambda\frac{z^2\left[I_{mk}^{\mu}(\alpha,\beta)f(z)\right]''}{I_{mk}^{\mu}(\alpha,\beta)f(z)}\right\}<\beta. \tag{2}$$

Remark 1.2: If we set $I_{mk}^{\mu}(\alpha,\beta)=1$ and $\lambda=0$ in (2), then it reduces to the class $S^*(\alpha,\beta)$. It is clear that $S^*(\alpha,\beta) \subset S^*(\alpha)$ and $S^*(\alpha,\beta) \subset M(\beta)$.

Definition 1.3: Let $\lambda \geq 0$ *and* $0 \leq \alpha<1<\beta$, we denote by $K_{\Omega,\Sigma}^{\theta}\left(\lambda;\alpha;\beta\right)$ the class of bi-univalent functions consisting of the functions in A such that

$$f \in K_{\Omega}^{\theta}\left(\lambda;\alpha;\beta\right) \text{ and } f^{-1} \in K_{\Omega}^{\theta}\left(\lambda;\alpha;\beta\right), \tag{3}$$

where f^{-1} is the inverse function of f.

Remark 1.4: If we set $\lambda=0$ in (3), for simplicity, we write $S_{\Omega,\Sigma}^{\theta}*(\alpha,\beta)$ instead of $K_{\Omega,\Sigma}^{\theta}\left(0;\alpha;\beta\right)$.

In this paper, we introduce new subclasses of $K_{\Omega}^{\theta}\left(\lambda;\alpha;\beta\right)$. We find the bounds for the coefficients of the functions in the class $K_{\Omega,\Sigma}^{\theta}\left(\lambda;\alpha;\beta\right)$. The techniques used in this paper are similar to those introduced in Xu et. al (2012).

2 PRELIMINARY RESULT

Kuroki and Owa (2011) defined an analytic function $p:U \to \mathbb{C}$ by

$$p(z)=1+\frac{(\beta-\alpha)i}{\pi}\log\left(\frac{1-ze^{2\pi(1-\alpha)i/(\beta-\alpha)}}{1-z}\right),$$

$$\left(0 \leq \alpha<1<\beta; z \in U\right) \tag{4}$$

and they proved that p maps U onto the convex domain

$$\Phi=\left\{w:\alpha<R(w)<\beta\right\}.$$

Observe that the function p, define by (4), has the representation

$$p(z)=1+\sum_{n=1}^{\infty}B_n z^n,\ \left(z \in U\right), \tag{5}$$

where

$$B_n=\frac{(\beta-\alpha)i}{n\pi}\left(1-e^{2n\pi(1-\alpha)i/(\beta-\alpha)}\right),\left(n \in \mathbb{N}\right). \tag{6}$$

In order to prove our main results, we need the following lemmas.

Lemma 2.1 (Keorgh and Merkes, 1969)
Let $p(z)=1+c_1z+c_2z^2+\dots$, be a function with positive real part in U. Then, for any complex number v,

$$\left|c_2-vc_1^2\right|\le 2\max\{1,|1-2v|\}.$$

The proof of the next lemma is similar to that of Lemma 1.3 in Kuroki and Owa (2011).

Lemma 2.2
Let $f\in \mathrm{A}$ and $0\le\alpha<1<\beta$, then $f\in K_{\Omega}^{\theta}(\lambda;\alpha;\beta)$ if and only if

$$\frac{z\left[I_{mk}^{\mu}(\alpha,\beta)f(z)\right]'}{I_{mk}^{\mu}(\alpha,\beta)f(z)}+\lambda\frac{z^2\left[I_{mk}^{\mu}(\alpha,\beta)f(z)\right]''}{I_{mk}^{\mu}(\alpha,\beta)f(z)}\prec p(z),(z\in U) \qquad (7)$$

where $p(z)$ is given by (4).

Lemma 2.3 (Rogosinki, 1943)
Let $p(z)=\Sigma_{n=1}^{\infty}C_nz^n$ be analytic and univalent in U and suppose that $p(z)$ maps U onto a convex domain. If $q(z)=\Sigma_{n=1}^{\infty}A_nz^n$ is analytic in U and satisfies the subordination:

$$q(z)\prec p(z),(z\in U)\ \text{ then}\left|A_n\right|\le\left|C_1\right|,(n=1,2,\dots).$$

3 MAIN RESULT

We begin by presenting some coefficient problem involving functions of class $K_{\Omega}^{\theta}(\lambda;\alpha;\beta)$.

Theorem 3.1. *If $f\in K_{\Omega}^{\theta}(\lambda;\alpha;\beta)$, then*

$$|a_2|\le\frac{|B_1|}{(2\lambda+1)\Omega_k^m\theta_k^m}\quad\text{and}$$

$$|a_n|\le\frac{|B_1|}{(n-1)(n\lambda+1)\Omega_k^m\theta_k^m}\prod_{k=2}^{n-1}\left(1+\frac{|B_1|}{(k-1)(k\lambda+1)\Omega_k^m\theta_k^m}\right)$$

$(n=3,4,5\dots)$,

where $|B_1|,\Omega_k^m$ and θ_k^m is given by

$$|B_1|=\frac{2(\beta-\alpha)}{\pi}\sin\frac{\pi(1-\alpha)}{\beta-\alpha}, \qquad (8)$$

$$\Omega_k^m=\left[\left(\frac{\prod_{j=1}^{q}\Gamma\left(\alpha_j+A_j(k-1)\right)}{\prod_{j=1}^{s}\Gamma\left(\beta_j+B_j(k-1)\right)}\right)\left(\frac{1}{(k-1)!}\right)\right]\text{ and}$$

$$\Theta_k^m=\left[\frac{(k-1)^k(k+\mu-2)!}{\mu!(k-2)!}\right],m\in\mathbb{N}_0=\mathbb{N}\cup\{0\},\ k\ge 2,\ \mu\ge 0.$$

Proof.
Let us define

$$q(z)=\frac{z\left[I_{mk}^{\mu}(\alpha,\beta)f(z)\right]'}{I_{mk}^{\mu}(\alpha,\beta)f(z)}+\lambda\frac{z^2\left[I_{mk}^{\mu}(\alpha,\beta)f(z)\right]''}{I_{mk}^{\mu}(\alpha,\beta)f(z)},$$

$(z\in U)$ (9)

and let the function p be given by (4). Then, the subordination from Lemma 2.2 can be written as follows:

$$q(z)\prec p(z),(z\in U) \qquad (10)$$

Note that the function p defined by (4) is convex in U and has the form

$$p(z)=1+\sum_{n=1}^{\infty}B_nz^n,(z\in U)$$

where B_n is given by (6). If we let

$$q(z)=1+\sum_{n=1}^{\infty}A_nz^n,(z\in U)$$

Then from lemma 2.3 we see that the subordination (10) implies

$$|A_n|\le|B_1|,(n=1,2,\dots), \qquad (11)$$

where $|B_1|$ is given by (8). Now, (9) implies that

$$z\left[I_{mk}^{\mu}(\alpha,\beta)f(z)\right]'+\lambda z^2\left[I_{mk}^{\mu}(\alpha,\beta)f(z)\right]''=I_{mk}^{\mu}(\alpha,\beta)f(z)q(z),(z\in U).$$

Then, by comparing the coefficients of z^n on both sides, we see that

$$|a_n|=\frac{1}{(n-1)(1+n\lambda)\Omega_k^m\theta_k^m}\times\left(A_{n-1}+a_2A_{n-2}\,\Omega_k^m\theta_k^m+a_3A_{n-3}\,\Omega_k^m\theta_k^m+\dots+a_{n-1}A_1\,\Omega_k^m\theta_k^m\right).$$

A simple calculation together with the inequality (11) yields that

$$|a_n|=\frac{1}{(n-1)(1+n\lambda)\Omega_k^m\theta_k^m}\times\left|A_{n-1}\Omega_k^m\theta_k^m+a_2A_{n-2}\,\Omega_k^m\theta_k^m+a_3A_{n-3}\,\Omega_k^m\theta_k^m+\dots+a_{n-1}A_1\,\Omega_k^m\theta_k^m\right|$$

$$\le\frac{1}{(n-1)(1+n\lambda)\Omega_k^m\theta_k^m}\times\left|\left(A_{n-1}+\left[a_2A_{n-2}+a_3A_{n-3}+\dots+a_{n-1}A_1\right]\right)\Omega_k^m\theta_k^m\right|$$

$$\le\frac{1}{(n-1)(1+n\lambda)\Omega_k^m\theta_k^m}\times\left[\left(|A_{n-1}|+|a_2||A_{n-2}|+|a_3||A_{n-3}|+\dots+|a_{n-1}||A_1|\right)\Omega_k^m\theta_k^m\right]$$

$$\le\frac{|B_1|}{(n-1)(1+n\lambda)\Omega_k^m\theta_k^m}\sum_{k=1}^{n-1}|a_k|\Omega_k^m\theta_k^m$$

where $|B_1|$ is given by (8) and $|a_1|=1$. Hence, we have

$$|a_2|\le\frac{|B_1|}{(2\lambda+1)\Omega_k^m\theta_k^m}.$$

To prove the remaining part of the theorem, we need to show that

$$\frac{|B_1|}{(n-1)(n\lambda+1)\Omega_k^m\theta_k^m}\sum_{k=1}^{n-1}|a_k|\Omega_k^m\theta_k^m$$
$$\le \frac{|B_1|}{(n-1)(n\lambda+1)\Omega_k^m\theta_k^m}\prod_{k=2}^{n-1}\left(1+\frac{|B_1|}{(k-1)(k\lambda+1)\Omega_k^m\theta_k^m}\right)$$
$$n=3,4,5,\ldots \quad (12)$$

We use induction to prove (12). The case $n=3$ is clear. Next, assume that the inequality (12) holds for $n=m$. Then, a straightforward calculation gives

$$|a_{m+1}|\le \frac{|B_1|}{m\left[(m+1)\lambda+1\right]\Omega_k^m\theta_k^m}\sum_{k=1}^{m}|a_k|\Omega_k^m\theta_k^m$$
$$=\frac{|B_1|}{m\left[(m+1)\lambda+1\right]\Omega_k^m\theta_k^m}\left[\sum_{k=1}^{m=1}|a_k|+|a_m|\right]\Omega_k^m\theta_k^m$$

$$\le \frac{|B_1|}{m\left[(m+1)\lambda+1\right]\Omega_k^m\theta_k^m}\prod_{k=2}^{m-1}\left(1+\frac{|B_1|}{(k-1)(k\lambda+1)}\right)\Omega_k^m\theta_k^m$$
$$+\frac{|B_1|}{m\left[(m+1)\lambda+1\right]\Omega_k^m\theta_k^m}$$
$$\times\frac{|B_1|}{(m-1)(m\lambda+1)}\prod_{k=2}^{m-1}\left(1+\frac{|B_1|}{(k-1)(k\lambda+1)}\right)\Omega_k^m\theta_k^m$$
$$=\frac{|B_1|}{m\left[(m+1)\lambda+1\right]\Omega_k^m\theta_k^m}\prod_{k=2}^{m-1}\left(1+\frac{|B_1|}{(k-1)(k\lambda+1)}\right)\Omega_k^m\theta_k^m,$$

which implies that the inequality (12) holds for $n=m+1$.

Hence, the desired estimate for $|a_n|$ $(n=3,4,5,\ldots)$ follows, as asserted in theorem 3.1. This completes the proof of Theorem 3.1. ☐

Taking λ = 0 in Theorem 3.1, and using the identity

$$\frac{|B_1|}{n-1}\prod_{k=2}^{n-1}\left(1+\frac{|B_1|}{(k-1)}\right)\Omega_k^m\theta_k^m=\prod_{k=2}^{n}\left(\frac{k-2+|B_1|}{k-1}\right)\Omega_k^m\theta_k^m$$
$$(n=3,4,5,\ldots)$$

we obtain the following corollary.

Corollary 3.2. If $f\in S^*(\alpha;\beta)$, then

$$|a_n|\le \frac{|B_1|}{(n-1)\Omega_k^m\theta_k^m}\prod_{k=2}^{n-1}\left(1+\frac{|B_1|}{(k-1)\Omega_k^m\theta_k^m}\right)$$
$$\le \prod_{k=2}^{n}\left(\frac{k-2+|B_1|}{(k-1)\Omega_k^m\theta_k^m}\right),\ (n=2,3,4\ldots),$$

$|B_1|$ is given by (8).

By setting $I_{mk}^{\mu}(\alpha,\beta)=1$ in Corrolary 3.3 we have the result obtained by Sun et. al (2015).

Corollary 3.3. If $f\in S^*(\alpha;\beta)$, then

$$|a_n|\le \frac{|B_1|}{(n-1)}\prod_{k=2}^{n-1}\left(1+\frac{|B_1|}{(k-1)}\right)$$
$$\le \prod_{k=2}^{n}\left(\frac{k-2+|B_1|}{(k-1)}\right),\ (n=2,3,4\ldots),$$

$|B_1|$ is given by (8).

Remark 3.4. *If* $0\le\alpha<1<\beta,$ we have

$$|B_1|=\frac{2(\beta-\alpha)}{\pi}\sin\frac{\pi(1-\alpha)}{\beta-\alpha}$$
$$\le\frac{2(\beta-\alpha)}{\pi}\times\frac{\pi(1-\alpha)}{\beta-\alpha}$$
$$=2(1-\alpha)\le 2,$$

thus, from Corrolary 3.3 we obtain

$$|a_n|\le \prod_{k=2}^{n}\left(\frac{k-2+|B_1|}{(k-1)}\right)\le\prod_{k=2}^{n}\left(\frac{k}{k-1}\right)=n,$$
$$(n=2,3,4\ldots),$$

shows that how the coefficient bounds in Corrolary 3.3 are related to the well-known Bierberbach conjecture proved by Branges L. De (1985).

4 CONCLUSION

There are many function of univalent and bi-univalent function in analytic that can be concluded in this research. New subclasses of function $K_{\Omega,\Sigma}^{\theta}(\lambda;\alpha;\beta)$ of bi-univalent in open unit disc $U=\{z\in\mathbb{C}:|z|<1\}$ involving Fox-Wright and Kuroki-Owa functions have been studied.

The bounds for the coefficients of the functions in the class $K_{\Omega,\Sigma}^{\theta}(\lambda;\alpha;\beta)$ we obtained in this research as assert in Theorem 3.1 and Corrolary 3.2. If $I_{mk}^{\mu}(\alpha,\beta)=1$ the results obtained will have a connection with previous studies done by Sun et. al (2015).

The result of this research can be as guided and to make a remark on the next future research.

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